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Book Review: The Fokker–Planck Equation, Methods of Solutions and Applications

The Fokker–Planck Equation, Methods of Solution and Applications. H. Risken, Springer, New York, 1984, pp. 454.

The Fokker–Planck (F–P) equation is central to much of contemporary research on fluctuations in physical systems. Therefore, a monograph on the subject is one that should draw a warm welcome from researchers in related areas of statistical physics. This monograph contains a large body of valuable material on the F–P equation, but is unfortunately written in a monotone which does not allow the reader to distinguish between more and less important topics. Occasionally the treatment of material is baffling. For example, the author derives the F–P equation by starting from the Kramers–Moyal expansion, then invoking Pawula's theorem, that only a truncation at the second term will lead to solutions that are nonnegative. One misses the traditional condition on the infinitesimal moments that is the proper motivation for truncating the series at the second derivative term and is left wondering why the F–P equation should ever yield a physically meaningful approximation. There is a very elaborate discussion of changes of variable in the F–P equation, but I suspect that any investigator wanting to change variables would be able to do it without the machinery of tensors.

Most of the standard techniques based on the separation of variables are discussed here, and numerical methods are mentioned, but scarcely in enough detail to guide someone new to the area. The Feller classification of boundaries is mentioned, but clearer discussions of this subject are available, e.g., in the book by Gardiner. A clear introduction is given to the subject of Kramer's theory of reaction rates as well as for first passage times for processes described by the F–P equation. Considerable space is devoted to the solution of tridiagonal recurrence relations arising from an analysis of the master equation or in the discretization of the F–P equation. This leads to the application of continued fraction techniques, a subject area which the author has done much to develop. It is not clear from the discussion how accurate approximations based on truncation of the con-

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tinued fraction might be, but some comment would be most useful. The final chapters are devoted to Brownian motion in periodic potentials and statistical problems related to lasers, both of which fields have been enriched by the author's original contributions.

In summary, this is not the book to be used as an introduction to the F–P equation. For the reader with some acquaintance with the subject, it provides a compendium of possibly useful techniques which are not discussed in other available texts. A noticeable drawback is that there is little accompanying insight into how accurate these techniques may be, and the circumstances in which they should be tried.

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